Mathematical Models for Cathodic Protection of an Underground Pipeline with Coating Holidays: Part 1 — Theoretical Development

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ABSTRACT

Mathematical models were developed to predict cathodic protection (CP) requirements for coated pipelines protected by parallel anodes. This work was motivated by the need to estimate current and potential distribution on a pipe when anodes are placed nearby or when discrete coating holidays expose bare steel. The mathematical model solves Laplace’s equation for potential with boundary conditions appropriate for the pipe being protected, the anode, and any region through which current does not pass. The current density on bare steel was assumed to be composed of contributions from corrosion, reduction of dissolved oxygen, and evolution of hydrogen. Kinetic parameters were obtained from independent experiments. The anode was assumed to have a constant potential, and current was allowed to flow through the coating under the assumption that the coating is a high-resistance ionic conductor. A boundary element technique coupled with Newton-Raphson iteration was used to solve the governing equations for two- (2-D) and three-dimensional (3-D) configurations. Results showed good agreement with experimental values and can be used to assess viability of CP designs.

KEY WORDS: anodes, boundary elements, cathodic protection, coatings, films and film formation, holidays, modeling, pipelines and line pipe steel, polarization curves

INTRODUCTION

External corrosion on pipelines can be mitigated by coating the pipe with a high-resistance film. Cathodic protection (CP) is used to protect portions of the pipe that are coated inadequately or where the coating has degraded. Defects in the coating are termed holidays, and such holidays can expose bare steel. While few coatings will remain holiday-free during their service lives, the effect of the coating is to reduce the current requirement for a CP system since a much smaller portion of the bare pipe is exposed. In a conventional impressed-current CP system for a pipe, multiple anode groundbeds are placed at discrete sites along the pipeline. The anode groundbeds normally are located sufficiently far from the pipe that they can be considered remote. The remote anode location allows protection of a larger section of the pipeline with a single source.

However, there are instances where the conventional approach to CP design cannot be used. Anode placement near the pipeline might be preferred in regions with a restricted right-of-way or for pipelines buried in frozen soil. As the temperature of the pipe would be well above the freezing point of water in such cases, a zone containing melted water is formed around the pipeline that has an effective soil resistivity much lower than the frozen ground. This melted zone surrounding the pipe is called a thaw bulb.

Conventional anode resistance formulas were developed for bare pipes protected by remote anodes. Under these conditions, the current density at the
anode is much larger than on the pipe, and resistance formulas that ignore current and potential distribution around the pipe can be used. Current and potential distribution on the pipeline, however, must be considered for pipes with coating holidays or with anodes placed in close proximity to the pipe. In high-resistivity soil, the ohmic potential drop between the anode and the portion of the pipe furthest from the anode is significantly greater than that of the portion of the pipe closest to the anode. It is possible, therefore, to overprotect the portion of the pipe that is closest to the anode while underprotecting the part furthest from the anode. The underprotected region is subject to corrosion; whereas, hydrogen evolution can occur on the overprotected region. Large negative potentials should be avoided because hydrogen embrittlement or step-wise cracking, which is associated with hydrogen evolution, can cause failure of the pipe.

The need to consider current and potential distribution at the pipe when designing a CP system with anodes is due to questions about the nature of holidays and the way they are modeled. The current and potential distribution associated with holidays are ignored in traditional CP design for remote anode placements.\(^1,2\) The effectiveness of the coating appears in design calculations only as a factor reducing the current required to protect the structure. For example, a coating 90% effective reduces the current required to protect a structure to 10% of the current needed without the coating. The model of a holiday as having reduced the uniform coating efficiency is implicit in the usual method of designing CP systems by using anode-to-ground formulas, but this approach also ignores problems associated with potential distribution around the pipe circumference.

Newman presented design calculations that accounted for potential distribution around the pipe under the assumption that holidays could be considered as having reduced the coating efficiency.\(^3\) Assumption of a uniform reduced efficiency coating is justified when coating resistivity is reduced uniformly (e.g., by water uptake) or if holidays are very small (e.g., "pinhole" defects) and are distributed uniformly around the pipe. Brichau and Deconinck presented a model using the boundary element (BEM) and finite element (FEM) methods for buried pipe networks that are protected cathodically by anode beds.\(^4\) Their method of surface discretization assumed cylindrical or "pipe" elements with a uniform radial current density distribution. While the authors claimed the technique introduced only a "controllable and minor error" in the calculations, the assumption of a uniform current density distribution is valid only for a perfectly coated or for a fully bare pipeline. Large defect areas in the coating, however, may be created during shipment of the pipe to the installation site, lifting of the pipe into the ditch, and backfill operations. Experimental and modeling work by Kennelley, et al., has shown that the size of a holiday has a critical effect on performance of a CP system.\(^5\) Orazem, et al., demonstrated that the CP requirements for a coated pipe with discrete holidays are much more severe than for pipes with coating resistivities reduced by an amount corresponding to the holiday size.\(^6\) While the finite element model developed by Kennelley, et al.,\(^7\) and Orazem, et al.,\(^6\) treats discrete holidays, this model was restricted to two dimensions and assumed a linearized polarization curve for bare steel.

The present work was motivated by the need for an accurate, user-friendly model for CP that could account for the influence of discrete coating holidays in a parallel pipe-anode configuration.

**MATHEMATICAL MODEL**

The objective of the present work was to develop 2-D and 3-D models that would run on a personal computer (PC). The programs were written to operate under Microsoft Windows\(^8\) environments. Mathematical models (2-D and 3-D) were developed to predict current and potential distribution on an underground pipeline under a parallel anode CP system. The models were designed to study the effect of discrete coating holidays/defects of various sizes on the performance of the CP system.

**Mathematical Development**

Under the assumption that concentrations are uniform, the current and potential distribution in the soil is governed by Laplace's equation, which can be written in rectangular coordinates as:

$$ \nabla^2 \Phi = \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0 $$

To solve this equation, boundary conditions are used to describe relationships between current and potential that are appropriate for any surfaces within the CP system.

*Insulators* — At insulating surfaces such as the periphery of the thaw bulb, the current in the direction normal to the surface is equal to zero. Thus, the boundary condition applied to the insulating surface was that the potential gradient perpendicular to the surface is equal to zero.

*Sacrificial Anodes* — The potential of the sacrificial anodes was assumed to be fixed. Some typical values have been reported by Peabody.\(^1\) The use of a fixed anode potential does not account for the potential shift that occurs when the current density at the

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1. Trade name.
anode is large. In this work, the shift in potential was treated by assuming a more positive potential for the anode (e.g., 1.7 V vs a copper-copper sulfate electrode [Cu-CuSO₄] instead of the nominal 1.8 V_{Cu-CuSO₄} for a high-performance Mg anode). The assumption of a constant potential could be relaxed by including a kinetic polarization equation such as described for the bare portion of the pipe.

**Bare Portion of Pipe (Holiday)** — Electrochemical reactions that can occur on bare steel include corrosion (oxidation of the metal):

\[ \text{Fe} \rightarrow \text{Fe}^{2+} + 2e^- \]  
(2)

reduction of dissolved oxygen:

\[ \text{O}_2 + 2\text{H}_2\text{O} + 4e^- \rightarrow 4\text{OH}^- \]  
(3)

and evolution of hydrogen:

\[ \text{H}_2\text{O} + 2e^- \rightarrow \text{H}_2 + 2\text{OH}^- \]  
(4)

The total current density (i) may be given by:

\[ i = -\kappa \nabla \Phi \]  
(5)

or, in terms of the contributions from individual reactions:²⁻¹¹

\[ i = 10 \left( \psi - E_{Fe} \right) \beta_{Fe} - \frac{1}{10 \left( \psi - E_{O_2} \right) \beta_{O_2}} - 10 \left( \psi - E_{H_2} \right) \beta_{H_2} \]  
(6)

where the first term represents the corrosion reaction, the second term represents oxygen reduction (which can be mass-transfer limited), and the third term represents hydrogen evolution. The parameters E_{Fe}, E_{O_2}, and E_{H_2} are "effective" equilibrium potentials that include the influence of the exchange current density. The parameters β_{Fe}, β_{O_2}, and β_{H_2} are the Tafel slopes for the respective reactions, and i_{O_2} is the mass-transfer limited current density for oxygen reduction. The potential used in this equation is:

\[ \psi = V_{pipe} - \Phi_0 \]  
(7)

the potential of the pipe minus the potential of a reference electrode located in the electrolyte adjacent to the pipe. The parameters needed were obtained by fitting Equation (6) to polarization data obtained at the Plano, Texas, facilities of ARCO Oil and Gas.²⁻¹² In Equation (6), a cathodic current is assumed to be negative.

**Coated Portion of the Pipe** — Under the assumption that the coating is a high-resistance ionic conductor and that the electrochemical reaction takes place on the surface of the steel pipe, the overall potential drop will be the sum of potential drops through the film and across the steel-film interface. When the coating resistance is much larger than the polarization resistance associated with the electrochemical reactions at the steel-coating interface, the relationship between current and potential is given by:

\[ i_n = \frac{1}{\rho_{lim} \delta_{lim}} (\psi - V_{corr}) \]  
(8)

where \( \rho_{lim} \) and \( \delta_{lim} \) are the film resistivity and thickness, respectively, and \( V_{corr} \) is the value of \( \psi \) at the corrosion potential. The polarization of the steel beneath the coating would need to be considered if the effective resistance of the coating is small.

**Method for Solution**

Analytic solutions to this equation are possible for simple geometries and are the basis for the anode resistance formulas routinely applied to CP design. While the geometry for the parallel anode impressed-current system studied here is relatively simple, the added complexity associated with holidays in the pipe coating necessitate use of a numerical method. The numerical methods that could be used include finite element, finite difference, and boundary element techniques. Finite element techniques (used in previous work⁵⁻⁶) are very flexible since nodes can be concentrated in regions of high current density and can be used to map curved as well as rectangular surfaces. A large number of nodes are needed, however, to obtain accurate results. Therefore, finite element techniques are difficult to apply on a PC. In boundary element techniques, Green's theorem is used to reduce the dimensionality of the problem (i.e., a 2-D problem is converted to a 1-D problem or a 3-D problem is converted to a 2-D problem). A significant savings in computational speed and memory requirements is achieved.

A boundary element technique was used to solve Laplace's equation for potential subject-to-boundary conditions that describe insulating and metal surfaces. The application of the BEM to potential problems has been discussed extensively in literature (e.g., references 7 through 11 and 13 through 28). The integral equation for this technique relates the potential anywhere in the domain to the potential and potential derivatives on the surface. This approach is especially relevant to corrosion engineering problems such as CP because conditions at the pipe or anode surfaces are the major concerns of the corrosion engineer. In addition, the reduction in dimensionality results in savings of the limited memory resources of the PC. The technique, however, is still numerically intensive. Evaluation of the integral equation requires discretization of the domain or boundary and leads
to a numerical solution of the problem. The nonlinear boundary condition that describes the kinetics at the surface of bare steel leads to an iterative solution that may be solved using an iterative Newton-Raphson approach.\textsuperscript{11,23,28}

**Governing Equations for the BEM**

The BEM implements the integral equation given by:

\[
c_i u_i + \int_{\Gamma} uq'd\Gamma = \int_{\Gamma} qu'd\Gamma \tag{9}
\]

which relates the potential \( u \), at point \( i \) anywhere in the domain to the potential \( u \) and flux \( q \) (i.e., \( q = \partial u / \partial n \)) on the boundary surfaces \( \Gamma \). The parameter \( c_i \) is a geometric constant. The notation used in this section is that used in the boundary-element literature (e.g., references 7 through 11 and 13 through 28). For a solution (or medium) of uniform bulk resistivity \( K \), the fundamental solution \( u^* \) for a 2-D geometry is given by:

\[
u^* = \frac{1}{2\pi} \ln \left( \frac{1}{r} \right) \tag{10}
\]

and for a 3-D domain is given by:

\[
u^* = \frac{1}{4\pi r} \tag{11}
\]

where \( r \) is the distance of the boundary element to the nodal point \( i \).

An element is comprised of nodes that define the shape and extent of the discretized boundary. The constant element is the simplest form, where the potential and flux are assumed to be uniform on the boundary element. Higher-order interpolating functions provide relationships between potential (or flux) and position and improve the accuracy of the calculations. In the 2-D models for example, the potential or flux for linear elements is given by: \textsuperscript{22,25}

\[
u(\xi) = \phi_1 u^1 + \phi_2 u^2 \tag{12}
\]

\[
q(\xi) = \phi_1 q^1 + \phi_2 q^2 \tag{13}
\]

where \( \xi \) is a dimensionless distance from +1 to −1, \( u^1 \) and \( q^1 \) are the nodal potential and flux, respectively, and the interpolating functions are given by:

\[
\phi_1 = \frac{1}{2} (1 - \xi) \text{ and } \phi_2 = \frac{1}{2} (1 + \xi) \tag{14}
\]

For 3-D problems, the surfaces can be discretized using triangular, rectangular, or cylindrical sections.\textsuperscript{4,13,15,25} Constant elements or interpolation functions also are used to define the approximate behavior of potential and flux at the boundary elements.

Discretization of the surfaces into boundary elements and the use of interpolating functions for potential \( u \) and flux \( q \) allow the numerical solution of the integral equation. The boundary condition imposed at the surface determines whether the potential or the flux is the unknown quantity. For an insulator surface, the flux \( q \) is equal to zero and the potential \( u \) is the unknown nodal quantity. For the anode assumed to be an equipotential surface, the flux \( q \) is the unknown quantity. Since the holiday (bare metal surface) uses a nonlinear boundary condition, the numerical approach yields a system of nonlinear equations where the initial guesses to the unknown variables are refined successively by an iterative Newton-Raphson technique.\textsuperscript{4,5,18,26}

**Computer Programs**

While the accuracy of the BEM calculations improved with a greater number of elements, the computational time required to obtain converged answers and the amount of hard disk space and random access memory (RAM) needed for the calculations increases nearly by the square of the number of elements. The choice of a discretization method was determined by the need for accurate converged answers and by the computational speed and limited hardware resources of the PC. Gridmakers were developed that automatically created the mesh for the 2-D and 3-D cell geometries. A large number of elements was assigned at and near the holidays, where the current densities are large, to improve the accuracy of the calculations.

**2-D Models** — The 2-D models used single- and multidomain approaches. The single-domain approach models the system as a homogeneous region where the soil resistivity is uniform. The use of multiple domains was motivated by the observation that concentration variations take place near holidays where the current density is large. The multidomain approach accounts for the lower solution resistivity near the holiday surface by dividing the thaw bulb into a bulk subregion and holiday subregions. The resistivity within each subregion is uniform, but the resistivity within the holiday subregions are lower than the bulk value. Laplace’s equation still holds for each subregion because the resistivity is homogeneous within each domain.

The linear element described above provided accurate converged answers for the 2-D models. The 2-D models were written as 16-bit Windows applications using Borland Pascal 7.0. The typical closure error between the calculated current at the pipeline and anodes was < 0.005% when the number of elements was > 100. On a 486DX2, 66 MHz PC, the computational time required to obtain converged answers for a grid with 100 elements...
was < 5 min and ~ 25 min for a grid with 300 elements.

3-D Model — The use of the multidomain approaches resulted in a modest 5% to 15% increase in the calculated CP current values compared to the single-domain approach. Based upon results of the 2-D models, the single-domain approach was deemed to be sufficient to provide reasonable predictions of a CP system in three dimensions.

The 3-D model used the boundary element technique using constant elements to solve Laplace’s equation. The software was written as a 32-bit Windows application using Watcom C/368 version 9.5. A large number of triangular elements was required to discretize the pipeline with a parallel, ribbon-anode CP system. An automated gridmaker generated 3-D meshes with 1,200 to 1,800 elements based upon user inputs. Calculations for the matrices associated with the BEM (the “H and G matrices” in the BEM literature) are a function only of cell geometry (i.e., the discretized nodes and elements of the surfaces of the CP system). Therefore, the BEM matrices need to be calculated only once for a given grid (or mesh) file. To study the trends of various CP design simulations, the 3-D model reads back the values of the calculated BEM matrices from the hard disk and performs only the Newton-Raphson algorithm using the new boundary conditions.

For a 486DX2, 66 MHz PC with 32 megabytes of RAM, the 3-D models required ~ 4 h to calculate the BEM matrices for a grid file with > 1,600 elements. The iterative Newton-Raphson algorithm needed ~ 2 h to obtain converged results. A considerable savings in computational time was realized if the large binary file for the BEM matrices was maintained on the hard disk. The typical closure error between the calculated current at the pipeline and anodes was < 0.05% when the number of elements was > 1,200.

RESULTS AND DISCUSSION

Comparison of these programs to experimental data and the application to prediction of CP performance in various scenarios have been presented elsewhere. The objectives of this work were to illustrate the precision of the calculations by comparison to known solutions of Laplace’s equation and to use the program to obtain a physical insight into the CP system.

**Benchmarking of Computer Programs**

Numerical methods for solving Laplace’s equation typically have difficulty with singularities such as the boundary between an insulator (where the normal current is equal to zero) and an electrode (where edge effects tend to make the current density very large). An extreme example would be the primary current distribution that yields an infinite current density at the edge of the electrode. The errors caused by singularities can be reflected in a mismatch of current integrated over the electrode with a singularity and the electrode without such singularities. For example, in the 2-D finite element code used previously to model the CP of pipes with coating holidays, the current integrated over the pipe was 17% to 28% smaller than the current integrated over the anode.

The analytic solution obtained by Moulton for a rectangular enclosure with arbitrarily placed electrodes provides a useful test to gauge the performance of a numerical algorithm and can show the extent to which the BEM code can be used to calculate current distributions that show sharp changes, such as expected at the boundary between bare and coated steel. The cell (Figure 1) is a unit cube where the “long” electrode is 1.0 unit long and the “short” electrode is 0.5 units long. The other surfaces are insulators. Region S2 was used for the single-plane symmetry approach.

The dimensionless resistance is obtained by integrating the current density over the electrode surface, i.e.:

\[
WdR = \frac{W}{\int \int n \cdot i \, dx \, dy}
\]

where \(W\) is the width of the electrode, \(\kappa\) is the conductivity of the electrolyte, \(V\) is the applied potential between the two electrodes, and \(n \cdot i\) is the current density normal to the electrode surface. The comparison between model calculations and the theoretical result obtained by Moulton is presented in Table 1.
The calculations agree with the theoretical value with errors of < 0.5%. The closure error, which reflects the extent to which the current integrated over the anode is equal to that integrated over the cathode, also is < 0.5%. Comparison to Moulton’s solution for the primary current distribution is a stringent test of the computer program. Generally, kinetic limitations cause the current to be more uniform and reduce the amount of the singularity.

**Current Distribution At a Holiday**

A schematic of a section of coated pipe with a parallel anode CP system is shown in Figure 2. Typical distributions of boundary elements used for the 3-D calculations are presented in Figures 3 and 4 for the pipe surface with circular and rectangular holidays, respectively. As seen in Figures 3(b) and 4(b), the number of elements was increased significantly in regions of large current density. For the 2-D calculations, the number of elements also was increased at the holiday and on the coated surface adjacent to the holidays.

Calculated values of potential are presented in Figures 3 and 4 for a coated pipe in 20 kΩ/cm resistivity soils protected by sacrificial zinc ribbon electrodes. The much higher calculated current density at the holidays compared to the coated surface leads to a more positive potential at the holiday, as reflected by the color scaling of the pipe surface in Figures 3 and 4. Thus, the level of protection at the holiday is lower than that which would be predicted for the coated pipe.

The influence of the increased current distribution at holidays also is seen in the results of the 2-D model. An example calculation of the current and potential distribution around the circumference of the pipe is given in Figure 5 for a 2-D cross section of the pipe with parallel anodes. Figure 5 is the result of a 2-D calculation in which the holiday is assumed to consist of a slit extending the length of the pipe, although qualitatively similar results are obtained for circular holidays. The pipe diameter was 1.22 m (4 ft), the width of the holiday was 1.27 cm (0.5 in.), and the electrolyte resistivity was assumed to be 5,000 Ω·cm. The anode was assumed to be a single high-performance Mg anode 0.3 m (1 ft) from the pipe at the opposite side of the pipe from the holiday. The potential is most positive at the holiday (180°). Almost all the current is delivered to the holiday. This result is seen more clearly in Figure 6, where the position scale is expanded. The edge effect on current is visible, and, while symmetry was not assumed, the symmetry of the potential and current distributions is evident. As shown in Figure 7, the current delivered to the holiday is over 200 times the average current density delivered to the pipe, and is over 10² times the current density delivered to the coated portion of the pipe.

**TABLE 1**

<table>
<thead>
<tr>
<th>Case No.</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetry</td>
<td>None</td>
<td>None</td>
<td>1-plane</td>
<td>1-plane</td>
</tr>
<tr>
<td>Nodes</td>
<td>98</td>
<td>160</td>
<td>56</td>
<td>121</td>
</tr>
<tr>
<td>Elements</td>
<td>92</td>
<td>316</td>
<td>95</td>
<td>216</td>
</tr>
<tr>
<td>WxR evaluated at the long electrode</td>
<td>1.2150</td>
<td>1.2084</td>
<td>1.2148</td>
<td>1.2192</td>
</tr>
<tr>
<td>WxR evaluated at the short electrode</td>
<td>1.2147</td>
<td>1.2214</td>
<td>1.2115</td>
<td>1.2231</td>
</tr>
<tr>
<td>Theoretical value for WxR</td>
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<td>1.22004</td>
<td>1.22004</td>
<td>1.22004</td>
</tr>
<tr>
<td>Error at the long electrode</td>
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<td>0.96%</td>
<td>0.43%</td>
<td>0.071%</td>
</tr>
<tr>
<td>Error at the short electrode</td>
<td>0.44%</td>
<td>0.11%</td>
<td>0.70%</td>
<td>0.25%</td>
</tr>
<tr>
<td>Closure error</td>
<td>0.025%</td>
<td>1.07%</td>
<td>0.27%</td>
<td>0.32%</td>
</tr>
</tbody>
</table>

**FIGURE 2.** Geometry of a 3-D coated pipe protected by parallel anodes. Planes of symmetry are used to section the geometry. Coating holidays may be placed at the top or bottom of the pipe.

The dramatic difference between the current delivered to the holiday compared to the coated portion of the pipe and the qualitative agreement between the edge effect shown in Figures 6 and 7 to that reported for a disk electrode suggests the pipe with a small coating holiday could be modeled as an active disk in an infinite insulating plane. In fact, for a circular holiday, a good approximation to the current requirement could be calculated by assuming the ohmic resistance is governed by the holiday and can be given by the primary resistance for a disk.
FIGURE 3. An example of the boundary element discretization for a pipe with a circular holiday: (a) the pipe surface and (b) a detailed view of the holiday region. The color pattern represents the calculated potential distribution for a 1.22-m (4-ft)-diam coated pipe with a 5.715-cm (2.25-in.)-diam holiday in soil with a 20-kΩ/cm resistivity protected by zinc ribbon sacrificial anodes.

FIGURE 4. An example of the boundary element discretization for a pipe with a rectangular holiday: (a) the pipe surface and (b) a detailed view of the holiday region. The color pattern represents the calculated potential distribution for a 1.22-m (4-ft)-diam coated pipe with a 61-cm (24-in.)-long holiday in soil with a 20 kΩ/cm resistivity protected by zinc ribbon sacrificial anodes.

electrode. A calculation using a polarization curve such as given in Equation (6) coupled with the equation for the primary resistance gives average current densities on the holiday that are very close to those calculated by the model. This approach is used to give a good initial guess for the iterative calculations of the model. A simple approach such as described here, however, fails when the holidays are large, does not provide information on the potential distribution around the circumference and along the length of the pipe, does not allow treatment of the effect of multiple holidays, and does not account for use of different anode materials. As shown elsewhere, the boundary element models provide an ability to assess the influence of typical CP design parameters.

Polarization and Ohmic Control of CP Current

The predicted current delivered to a circular holiday is presented in Figure 8 as a function of solution resistivity for different anode configurations. At high resistivities, the ohmic resistance is dominant and constrains the amount of current that can be delivered for a given potential driving force (assumed here to be delivered by a high-performance Mg anode). At lower electrolyte resistivities, the polarization
Resistance contributes significantly. This result is consistent with the observation that it is more difficult to provide adequate protection to a holiday in high-resistivity environments.

CONCLUSIONS

- The conventional anode resistance formulas that were developed for bare pipes protected by remote anodes cannot be used to describe the current and potential distribution on a pipeline with coating holidays or with anodes placed in close proximity to the pipe. The need to consider the current and potential distribution at the pipe when designing a CP system with anodes close to the pipe requires that holidays be modeled in a realistic way.
- The 2-D and 3-D boundary element mathematical models presented here provide the flexibility to model the performance of CP designs for a variety of parallel anode configurations. A unique feature of these models is the ability to perform calculations on a PC. The model offers a convenient tool to quantify the performance of a CP system and allows the user to determine the influence of relevant parameters (e.g., soil resistivity, coating damage, and anode type and spacing). The model also can be used as an educational tool to identify factors that control CP performance in different operating conditions.

ACKNOWLEDGMENTS

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FIGURE 8. CP current delivered to a coated pipe with a 5.67-cm (2.25-in.)-diam circular holiday calculated by a 2-D BEM model. Polarization resistance is negligible compared to the ohmic resistance at high electrolyte resistivities.