FILTRATION

Introduction

Filtration is a process of separation of solid particles from a liquid in which they are suspended by using a medium (filter) through which only the liquid can pass. In this lab, you will first perform experiments with a small-scale batch filter and use your experimental data to predict operating conditions for a large-scale continuous rotary drum filter commonly used in industrial applications. You will then verify your predictions by experiments with the drum filter. The experiments will be performed with a slurry of diatomaceous earth in water.

Objectives

The main learning objective is to gain experience in scaling up from a small-scale benchtop system to a large-scale production operation.

**Batch Filtration (Small Scale)**

1. Validate the theoretical predictions for dependence of the collected filtrate volume on time.
2. Determine effects of pressure drop and slurry concentration on the porosity and permeability of the filter cake.
3. Use results of the batch filtration experiment to predict dependence of the filtrate flow rate and the rate of the filter cake production in the large-scale continuous filtration on the speed of the drum rotation, pressure drop, and the slurry concentration.

**Continuous Filtration (Large Scale)**

1. Determine effect of the slurry concentration, pressure drop, and drum rotation speed on the filtrate flow rate and the rate of cake formation.
2. Compare your theoretical predictions based on the batch filtration data with experimental results for the continuous filtration.
Batch Filtration

**Controlling parameters:** Pressure drop, concentration of slurry.

**Performance indicator:** Dependence of the collected filtrate volume on time.

The batch filtration system consists of a graduated cylinder, a filter medium (a canvas cloth or a porous plate) fixed at the bottom of the cylinder, and a conical flask below the filter medium (see Figure 1-1). Feed (slurry) is poured into the graduated cylinder, the filtrate is collected in the conical flask, and the cake is accumulated on top of the filter medium.

The batch filtration can be performed in two regimes:

1. **Gravity-driven filtration.** The fluid flow from the cylinder into the conical flask is driven by gravity (i.e., the hydrostatic pressure of the slurry above the filter).
2. **Pressure-driven filtration.** A vacuum pump is attached to the conical flask and is used to create a constant vacuum pressure inside the flask. The main driving force for the filtrate flow is then the difference between the atmospheric pressure and the pressure in the flask. While the hydrostatic pressure still contributes to the driving force, it is usually much smaller than the vacuum pressure generated by the pump.

![Figure 1-1. Schematic of the Batch Filtration system.](image)

The objective of the batch experiment is to determine porosity (i.e., volume fraction of pores) and permeability of the cake. Once these values are obtained, they will be used to make predictions for the continuous filter. Below we briefly discuss the theories of the batch and continuous filtration. This discussion is based on Ref. [1].
Fluid flow through a porous material, such as the filter cake in this experiment, is described by Darcy’s law:

\[
\frac{\kappa \Delta p}{\mu L} = \frac{Q}{A}. \tag{1}
\]

Here, \( \kappa \) is the cake permeability, \( \mu \) is the fluid viscosity, \( \Delta p \) is the pressure drop across the cake, \( L \) is the cake thickness, \( Q \) is the volumetric flow rate of the liquid, and \( A \) is the area of the cake cross-section. Eq. (1) has a simple physical interpretation: the fluid flow rate is directly proportional to the driving force \( A \Delta p \) applied to push the fluid through the cake and is inversely proportional to the cake thickness. In batch filtration, the fluid flow is driven by (i) the hydrostatic pressure of slurry above the cake and (ii) the vacuum pressure (i.e., the difference between the atmospheric pressure and the pressure inside the flask located below the filter).

Eq. (1) indicates that, in addition to the cake thickness and the driving force, the flow rate depends on the material properties of the cake and the fluid (\( \kappa \) and \( \mu \), respectively). Permeability \( \kappa \) is the measure of the ability of a porous material to allow fluids to flow through. The SI unit of permeability is m\(^2\). Another common unit of permeability is darcy (d), \( 1 \text{d} \approx 10^{-12} \text{m}^2 \). To get an idea about expected order of magnitude of the cake permeability, note that permeability of sand is roughly \( 1 \text{d} \) [2].

Values of the water viscosity \( \mu \) at various temperatures are available in the literature and values of \( L, A, \) and \( Q \) can be measured directly. In particular, \( Q \) can be measured by the “bucket and stopwatch” method,

\[
Q = \frac{dV}{dt} \tag{2}
\]

Here, \( V(t) \) is the volume of the filtrate collected at time \( t \). Therefore, we can solve Eq. (1) for the only remaining unknown, namely the cake permeability \( \kappa \).

However, this approach is not practical due to time dependence of \( L, Q, \) and \( \Delta p \). The thickness \( L(t) \) of cake accumulated on the filter surface grows with time. This leads to an increasing resistance to the filtrate flow through the cake and, hence, a decreasing \( Q(t) \). Moreover, as filtration proceeds, the height \( H(t) \) of the slurry column above the cake decreases, which decreases \( \Delta p(t) \) and, hence, further decreases \( Q(t) \). Therefore, in order to apply Eq. (1) directly, one needs to simultaneously measure and record the values of \( L(t), Q(t), \) and \( H(t) \) at various instances of time \( t \). Furthermore, the “bucket and stopwatch” method of flow rate measurement is inherently very inaccurate for time-dependent \( Q(t) \).

A more convenient approach to measurement of \( \kappa \) is to perform a least-squares fit to experimental data for the volume \( V(t) \) of collected filtrate. This approach requires measurement on \( V(t) \) only. To apply this approach, it is necessary to obtain a theoretical prediction for \( V(t) \) discussed below.

Rewrite Darcy’s law as a differential equation for \( V(t) \) by substituting Eq. (2) into Eq. (1):

\[
\frac{\kappa A}{\mu L} \Delta p = \frac{dV}{dt}. \tag{3}
\]

The cake thickness \( L(t) \) of the cake and the volume \( V(t) \) of the filtrate collected at time \( t \) are related through the mass balance for solids. The mass of the solids in the cake is
$$m_d = (1 - \epsilon)LA \rho_{solid}.$$ \hspace{1cm} (4)

Here, $m_d$ is the dry mass of the cake, $\rho_{solid}$ is the density of dry solids, and $\epsilon$ is the porosity (volume fraction of pores) of the cake. Solids making up the cake are obtained by separation of these solids from some volume $V_i$ of the liquid,

$$m_d = c_o V_i.$$ \hspace{1cm} (5)

Here, $c_o$ is the concentration of solids in the slurry (unit mass of solids per unit volume of liquid) and the liquid volume is

$$V_i = V + \epsilon LA.$$ \hspace{1cm} (6)

The 1st and 2nd terms in the right-hand-side of Eq. (6) are the volume of the filtrate collected in the flask and the volume of the pores in the cake, respectively. In writing Eq. (6), we assumed that all cake pores are occupied by the liquid. Equating the right-hand sides of Eqs. (4) and (5) and substituting $V_i$ from Eq. (6), we obtain the following equation for the mass balance of the solids:

$$(1 - \epsilon)LA \rho_{solid} = c_o (V + \epsilon LA)$$ \hspace{1cm} (7)

Assuming that $\epsilon LA$ is negligible in comparison with $V$, solving Eq. (7) for $L$, and substituting the result into Eq. (3), we obtain:

$$\frac{dV}{dt} = KA^2 \frac{\Delta p}{V},$$ \hspace{1cm} (8)

where

$$K = \frac{\kappa}{\mu} \frac{(1 - \epsilon) \rho_{solid}}{c_o} = \text{const.}$$ \hspace{1cm} (9)

Eq. (8) can be easily solved if $\Delta p$ is constant. In this case,

$$V(t) = (2K\Delta pt)^{1/2}A.$$ \hspace{1cm} (10)

Therefore, a plot of $V$ vs. $t^{1/2}$ should be a straight line and the value of constant $K$ can be obtained from the slope of this line.

The cake porosity $\epsilon$ can be determined by measuring weights of the wet and dry cakes and using them to determine the volume fraction of water (and, hence, pores) in the wet cake. Once $\epsilon$ and $K$ are known, one can solve Eq. (9) to obtain the cake permeability $\kappa$.

Eq. (10) is valid only in the case of constant $\Delta p$. In reality, $\Delta p$ is time dependent,

$$\Delta p = \Delta p_{pump} + \rho_{slurry} gH(t),$$ \hspace{1cm} (11)

where $\Delta p_{pump}$ is the pressure drop generated by the vacuum pump, $\rho_{slurry}$ is the slurry density, and $H(t)$ is the height of the slurry column above the cake. Dependence of $H(t)$ on time is directly related to that of $V(t)$. Therefore, Eq. (8) can still be solved albeit the solution becomes more complex. In the pressure-driven experiments, $\Delta p_{pump} = \text{const}$ and, if the contribution of the hydrostatic pressure to $\Delta p$ is negligible, one can assume that $\Delta p = \text{const}.$
Accounting for Resistance of Filtration Medium

The derivation above assumes that the only source of resistance to the fluid flow is the filtration cake and neglects resistance due to the filter medium. This is a reasonable assumption if the filter medium is a canvas cloth. However, if the filter medium is a porous plate, the validity of this assumption needs to be checked by measuring the filter permeability. This can be done by performing the batch “filtration” experiment with pure water instead of slurry. In this case, Darcy’s law (1) is still applicable, except now $\kappa$ and $L$ refer to the permeability and thickness of the filter plate (not cake). Assuming that the applied pressure is constant, the flow rate of water through the filter plate remains constant. Therefore, the filter resistance can be obtained directly by fitting the plot of $V(t)$ vs. $t$ to a straight line and applying Eq. (1).

Once the filter permeability is known, we can compare the filter resistance, $R_f = L_f/\kappa_f$, to the cake resistance $R_c = L_c/\kappa_c$. Here, the subscripts $f$ and $c$ are used to denote properties of the filter plate and cake, respectively. When the filtration process is first started, the filter resistance is larger than the cake resistance (since $L_c$ is very small). However, if this initial period is very short and $R_f \ll R_c$ for the rest of the filtration process, the filter resistance can be neglected.

In case the filter resistance is not negligible, the derivation presented in the previous section should be modified. The total resistance of the cake and filter is $R = R_f + R_c$. Therefore, Darcy’s law (1) yields the following relationship between the flow rate and the pressure:

$$\frac{Q}{A} = \frac{1}{R_f} \frac{\Delta p}{\mu} = \frac{1}{R_f + R_c} \frac{\Delta p}{\mu} = \frac{1}{L_f/\kappa_f + L_c/\kappa_c} \frac{\Delta p}{\mu} \tag{12}$$

The material balance equation (7) is still valid, assuming that amount of water accumulated in the pores of the filter plate is negligible in comparison with the amount of filtrate $V$. Assuming, as above, that $\epsilon LA$ is also negligible in comparison with $V$ and combining Eqs. (7) and (12), we obtain the following generalization of Eq. (8):

$$\frac{dV}{dt} = \frac{A^2 \Delta p}{C + V/K} \tag{13}$$

where

$$C = A\mu L_f/\kappa_f = \text{const} \tag{14}$$

and $K$ is given by Eq. (9). Let us solve Eq. (13) in the case of constant $\Delta p$:

$$\frac{V^2}{2K} + CV = A^2 \Delta pt. \tag{15}$$

Comparing Eqs. (10) and (15), we see that the filter resistance yields an additional term, $CV$. 
Continuous Filtration

Controlling parameters: Pressure drop, concentration of slurry, drum speed.

Performance indicators: Filtrate flow rate, rate of cake formation.

Rotary drum filter consists of a drum rotating in a tub of slurry to be filtered (see Figure 1-2). As the drum rotates through the slurry, the vacuum pulls liquid and solids onto the drum. The liquid portion of the slurry is then pulled through the filter media into the drum and is pumped away. The solids adhere to the filter surface and are eventually pushed from the drum by compressed air. The rotating drum continuously takes in feed from the slurry tank and removes the cake form the surface after it emerges from the slurry. This establishes a continuous operation.

Figure 1-2. Rotary drum filter: (a) the Bird-Young filter installed in the Unit Operations Lab; (b) Schematics of the filter operation.

A rotary drum filter can be modeled using a modification of the theory for the pressure-driven batch filtration discussed in the previous section. This is possible because the continuous filtration process can be thought of as multiple batch processes being performed in parallel. To see this, let us divide the portion of the drum surface immersed into the slurry into \( N \) small segments. Each of these segments acts like a small batch filter: when a filter segment first enters the slurry, it has essentially no solids on its surface, just like a batch filter before filtration has started. As the filter segment travels through the slurry, it accumulates more and more cake on its surface. This increases resistance to the filtrate flow through the segment and reduces the filtrate flow rate, just like in batch filtration.

The volume of filtrate that passed through a filter segment after it spent time \( t \) in the slurry is given by the same equation (10) as for the batch filter\(^1\). In the case of the continuous filter, the filter area \( A \) in Eq. (10) should be replaced by the area \( A_s \) of a filter segment,

\[
A_s = \frac{A}{N} \tag{16}
\]

\(^1\) The filter resistance can be neglected for the continuous filter, since filter medium in this case is a canvas cloth.
Here, \( A \) is the total area of the portion of the filter drum immersed into the slurry. To determine this area, let us define the angle \( \theta \) as shown in Figure 1-2b. Then

\[
A = wR\theta,
\]  

(17)

where \( w \) and \( R \) are the drum width and radius, respectively.

The amount that of time spent by each filter segment in the slurry is

\[
\tau = \frac{\theta}{\Omega},
\]

(18)

where \( \Omega \) is the angular velocity of the drum (in rad/s). Substituting this time and the segment area \( A_s \) into Eq. (10), we obtain the following total volume \( V_s \) of filtrate that passes through a single filter segment while it travels through the slurry:

\[
V_s = V(\tau) = (2K\Delta p\tau)^{1/2}A_s.
\]

(19)

The constant \( K \) is given by Eq. (9). Therefore, the average flow rate of the filtrate through a filter segment is

\[
Q_s = \frac{V_s}{\tau} = \left(\frac{2K\Delta p}{\tau}\right)^{1/2} A_s
\]

(20)

Since there are \( N \) segments immersed in the slurry, the total flow rate of the filtrate through the drum surface is

\[
Q = NQ_s = \left(\frac{2K\Delta p}{\tau}\right)^{1/2} A.
\]

(21)

Substituting the expressions (17) and (18) for \( \tau \) and \( A \) into Eq. (21), we obtain

\[
Q = \left(\frac{2K\Delta p\Omega}{\theta}\right)^{1/2} wR\theta = (2K\Delta p\Omega\theta)^{1/2} wR.
\]

(18)

Once \( Q \) is known, we can easily obtain the cake production rate,

\[
\dot{m}_d = c_0 Q,
\]

(19)

where \( \dot{m}_d \) is the rate of production of dry solids (unit mass per unit time).

**References**