Two Rotameter Experiment

Theory

Rotameters are flowmeters that measure fluid flow rate using force balance between the gravity, buoyancy, and drag forces. They are reliable, relatively inexpensive, and require no external power sources to operate. A rotameter is a vertical conical tube with a float in it, as shown schematically in Figure 1. The cross-section area of the tube increases from bottom to top. The fluid flows through the tube and the float is free to move in the vertical direction inside the tube. The fluid flow rate is directly related to the location of the float, with larger flow rates corresponding to higher float positions.

![Figure 1. Schematics of a rotameter at (a) low and (b) high fluid flow rates. $F_d$, $F_g$, and $F_b$, and are the drag, gravity, and buoyancy forces, respectively.](image)

To see this, consider the balance of forces acting on the float. The sum of the gravity and buoyancy forces is

$$F_g + F_b = V_f (\rho_f - \rho) g,$$

where $V_f$ and $\rho_f$ are the volume and the density of the float, $\rho$ is the fluid density, and $g$ is the acceleration due to gravity. The drag force acting on the float is [1]

$$F_d = C_D \rho A_p \frac{v^2}{2},$$

where $C_D$ is the drag coefficient, $A_p$ is the area of the projection of the float onto the horizontal plane, and $v$ is the fluid velocity directly below the float. Due to the conical shape of the rotameter tube, the fluid velocity depends not only on the fluid flow rate $Q$, but also on the float position in the vertical direction, since the latter determines the cross-section area $A_s$ of the tube.

Therefore, for a given flow rate $Q$, the force balance
is achieved when the float reaches an equilibrium position such that the rotameter cross-section \( A_s \) satisfies Eq. (3). Increasing the flow rate leads to an increase of \( A_s \) necessary to satisfy the force balance. This, in turn, leads to the float rising in the tube, as illustrated in Figure 1.

Flow rates corresponding to different float positions are usually indicated on a scale located next to the rotameter tube. This calibration of the rotameter (i.e., determination of the relationship between the float position and the flow rate) is typically performed by a manufacturer for a specific fluid at specific operating conditions. For example, rotameters for gases are frequently calibrated to measure flow rate of air at the STP conditions (\( P = 1 \) atm, \( T = 25^\circ \text{C} \)). However, often one needs to use a rotameter for different fluids and/or at different operating conditions. Therefore, it is necessary to convert rotameter readings to the actual flow rates. In this experiment, you will consider gas rotameters operating at different pressures. Since gases are compressible, their density is affected by the pressure. Hence, according to Eq. (2), the drag force is also affected by the pressure.

Let us discuss how to obtain the actual flow rate \( Q_a \) from the rotameter reading \( Q_r \). Assume that the fluid is gas so that \( \rho \ll \rho_f \). Then the force balance Eq. (3) at the reference and actual operating conditions can be simplified as follows:

\[
C_D \rho_r A_p \left( \frac{Q_r}{A_s} \right)^2 = V_f (\rho_f - \rho) g
\]

and

\[
C_D \rho_a A_p \left( \frac{Q_a}{A_s} \right)^2 = V_f \rho_f g
\]

respectively. Here, \( \rho_r \) and \( \rho_a \) are gas densities at the reference and actual conditions, respectively. (The reference conditions are the conditions for which the rotameter was calibrated, i.e. at these conditions the actual flow rate is equal to the rotameter reading.) Dividing Eq. (4) by Eq. (5), we obtain

\[
\rho_r Q_r^2 = \rho_a Q_a^2
\]

This equation can now be easily solved to obtain \( Q_a \).

The 2-rotameter experiment in the Unit Operations Lab consist of two rotameters connected in series, with the lower rotameter connected to a compressed air source and the upper rotameter open to the atmosphere. Pressure in the lower rotameter is controlled by a pressure...
regulator near its inlet and the flow rate is controlled by a needle valve located between the rotameters. The needle valve also creates a pressure drop, which leads to different pressures in the two rotameters. This causes different readings on these rotameters, even though the mass flow rates are the same. Of course, the volumetric flow rates through these rotameters are different due to differences in pressure. However, the rotameter readings are not necessarily equal to the actual volumetric flow rates! In this lab, you will compare the experimental values of the mass flow rates, volumetric flow rates, and the readings of these two rotameters.

References